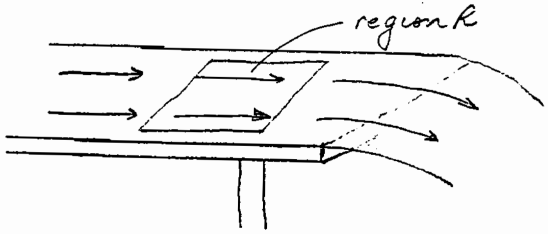
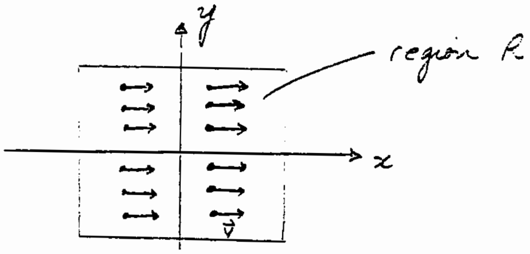
SOME EXAMPLES OF VECTOR FIELDS IN PHYSICS

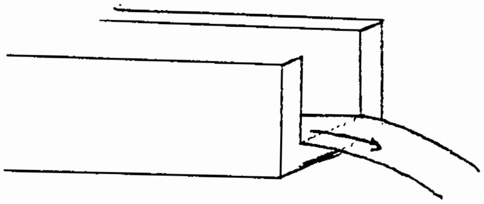
I. VELOCITY VECTOR FIELDS



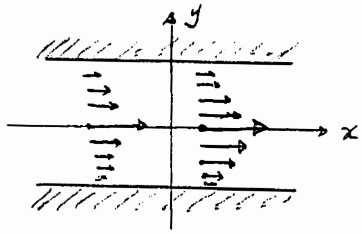
FLOW OF A THIN FILM OF WATER OVER A FLAT SURFACE AT CONSTANT VELOCITY v



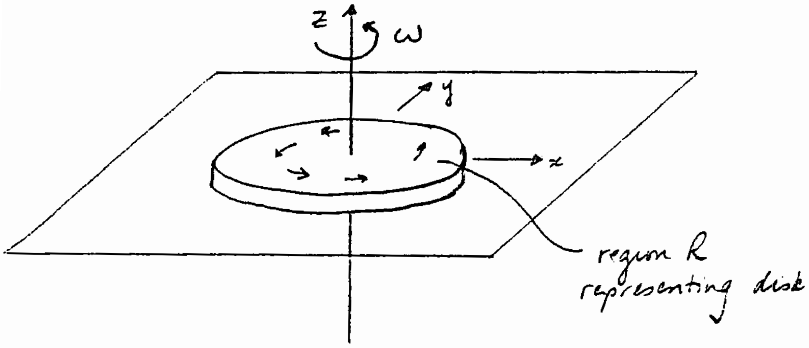
At each point $(x,y) \in R$, the vector $\vec{v} = v\hat{\imath}$ represents the velocity of a fluid particle.



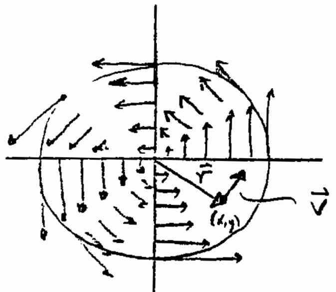
THIN FILM OF WATER FLOWING IN A TROUGH



WATER CLOSE TO WALLS TRAVELS SLOWER ("STICKING" AT WALL + FRICTION



A DISK ROTATING ABOUT ITS PRINCIPAL AXIS WITH ANGULAR FREQUENCY $\omega > 0$



At each point $(x,y) \in R$, the velocity vector $\vec{v}$ is perpendicular to the position vector $\vec{r} = x\hat{\imath} + y\hat{\jmath}$:

$$\vec{v} = -\omega y\hat{\imath} + \omega x\hat{\jmath}$$

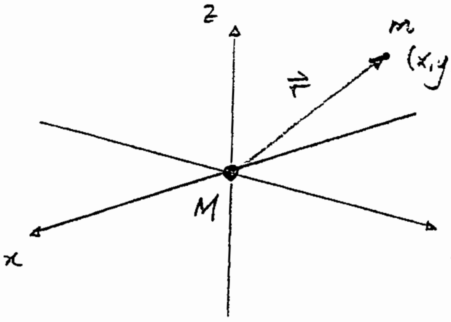
If we define $\vec{\omega} = \omega\hat{k}$, the angular velocity vector, then

$$\vec{v} = \vec{\omega} \times \vec{r}$$

Since $\vec{\omega} \perp \vec{r}$, speed $v = \|\vec{v}\| = \omega r$

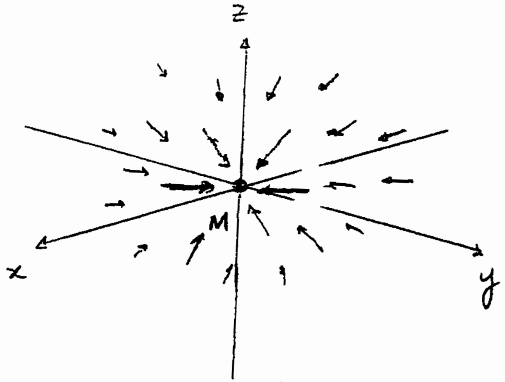
II. FORCE FIELDS

GRAVITATION



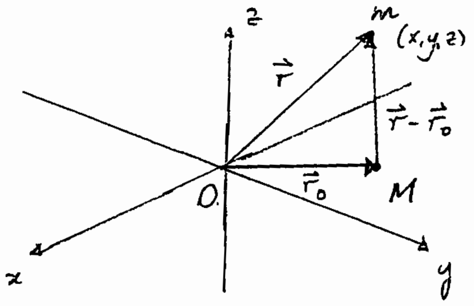
Force on mass m at $\vec{r}$ due to presence of M at 0

$$\vec{F}(\vec{r}) = -\frac{GMm}{r^3}\vec{r} = -\frac{GMm}{r^2}\hat{r}$$



Force field $\vec{F} = -\frac{GMm}{r^3}\vec{r}$ in $\mathbb{R}^3$

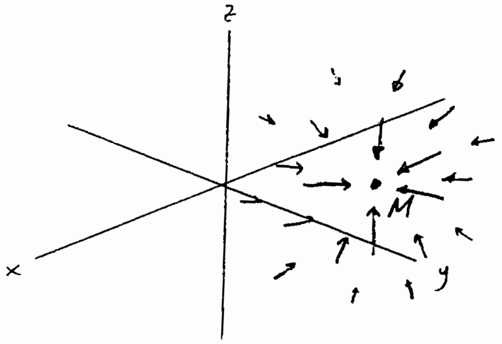
Note $\|\vec{F}\| = \frac{GMm}{r^2}$



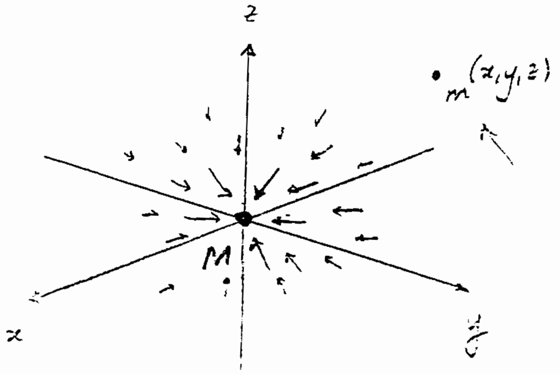
Force on mass m at $\vec{r}$ due to presence of M

at $\vec{r}_0 = (x_0, y_0, z_0)$

$$\vec{F}(\vec{r}) = -\frac{GMm}{\|\vec{r}-\vec{r}_0\|^3}(\vec{r}-\vec{r}_0)$$



Force field $\vec{F} = -\frac{GMm}{\|\vec{r}-\vec{r}_0\|^3}(\vec{r}-\vec{r}_0)$



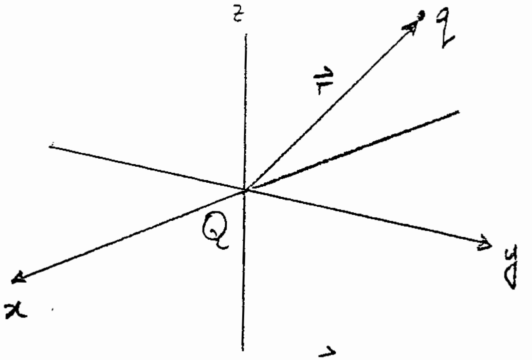
Force on mass m at $\vec{r}$

$$\vec{F} = m\vec{f}$$

Gravitational field (force per unit mass) at $\vec{r}$ due to presence of M at 0

$$\vec{f}(\vec{r}) = -\frac{GM}{r^3}\vec{r}$$

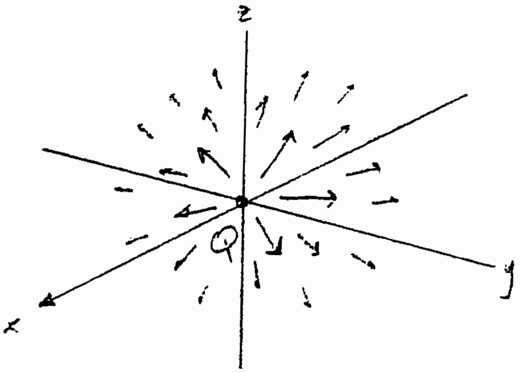
ELECTROSTATICS



Force on charge q at $\vec{r}$ due to presence of charge Q at O

$$\vec{F}(\vec{r}) = \frac{Qq}{4\pi\varepsilon_0 r^3}\vec{r} \qquad \text{(Coulomb's Law)}$$

If $Qq < 0$ (Q & q differ in sign) $\vec{F}$ is attractive
$Qq > 0$ (Q & q have same sign) $\vec{F}$ is repulsive



The electrostatic field (force per unit charge) at $\vec{r}$ due to the presence of (stationary) charge Q at O

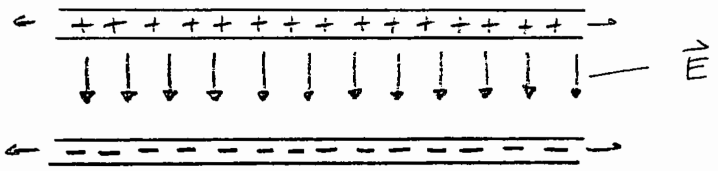
$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0 r^3}\vec{r}$$

In the picture, we have assumed that Q is positive. This does <u>not</u> imply that the force exerted by Q on a charge is repulsive.

Force exerted on q in electrostatic field $\vec{E}$ is

$$\vec{F} = q\vec{E}$$ If $q<0$, then $\vec{F}$ is attractive

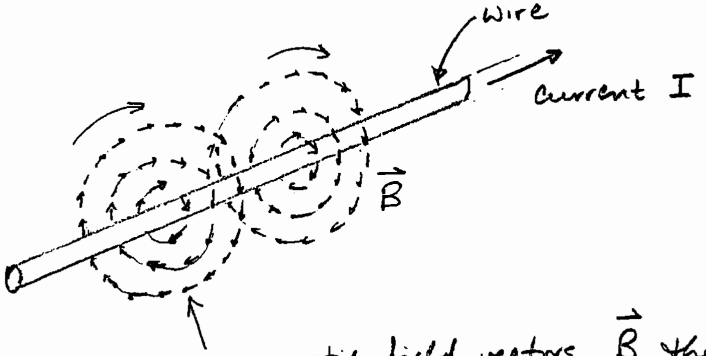
Electrostatic field $\vec{E}$ produced by two oppositely charged plates (ignoring edge effects) near each other:



Constant vector pointing from + plate to − plate at all points between them

ELECTROMAGNETISM

A moving charge generates a magnetic field:



magnetic field vectors $\vec{B}$ that encircle the current carrying wire (Biot-Savart law)