Further to Example 3.8, p. 39

$D: X \to Y \quad D = \frac{d}{dt} \quad X = \{ f \in C^1[0,1], f(0) = 0 \} \quad \| \|_0$

$Y = C[0,1] \quad \| \|_\infty$

$D^{-1}: Y \to X \quad (D^{-1}g)(t) = \int_0^t g(s)\,ds \quad g \in C[0,1]$

Let $g_n = \cos nx \quad n = 0, 1, 2, \ldots$

$\|g_n\|_\infty = 1$ (The g_n lie on the unit ball)

$$f_n = D^{-1} g_n \Rightarrow f_n(x) = \int_0^x \cos ns\,ds$$
$$= \frac{1}{n} \sin ns \Big|_0^x$$
$$= \frac{1}{n} \sin nx$$

$\|f_n\|_\infty = \frac{1}{n}$

$\Rightarrow \|f_n\|_\infty \to 0$ as $n \to \infty$

We conclude:

D^{-1} is <u>not</u> bounded from below, i.e. there exists no $m > 0$ s.t.

$$\|D^{-1} g\| \geq m \|g\| \quad \text{for all } g \in Y.$$

$\uparrow$ f_n, $\|f_n\|_\infty = \frac{1}{n} \to 0$ $\qquad$ $\uparrow$ g_n, $\|g_n\|_\infty = 1$