Addendum to p. 36

Alternate proof of L cts on $X \Rightarrow L$ is bounded.

L cts at $X \Rightarrow L$ cts at 0. Assume L is not bounded.

Then there exists a sequence $\{x_n\}$, s.t. $\|x_n\| = 1$

and $\|Lx_n\| \to \infty$ as $n \to \infty$

Define $y_n = \dfrac{x_n}{\|Lx_n\|}$ so that $\|y_n\| = \dfrac{\|x_n\|}{\|Lx_n\|} \to 0$ (1)

as $n \to \infty$

This implies that $y_n \to 0$. By continuity of L at 0,

it follows that $Ly_n \to L0 = 0$.

But $\|Ly_n\| = \left\| \dfrac{Lx_n}{\|Lx_n\|} \right\| = \dfrac{\|Lx_n\|}{\|Lx_n\|} = 1$

contradicting (1).

$\Rightarrow L$ is bounded.